RESEARCH DEPARTMENT

Calculation of overall line-up errors for PAL colour television distribution chains

RESEARCH REPORT No. PH-19

UDC 621. 397. 132:
519. 24

1968/25

THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION

RESEARCH DEPARTMENT

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SUMMARY

It is here assumed that errors in any one link of a distribution chain are distributed rectangularly, and that each such distribution can be regarded as discrete or quantized. It is also assumed that errors within individual links can be added (linearly) to obtain the overall error. The distribution of overall errors is calculated by convolution from the rectangular distribution for individual links. Convolution is equivalent to the algebraic multiplication of the 'generating functions' associated with the individual distributions. The derivation of the 'generating function' from a geometrical representation of a distribution and the reverse process are discussed.

Formulae and tables are derived for the convolution of up to five equal rectangular distributions, and it is shown how these results can be used to derive the convolution of unequal rectangular distributions in a straightforward manner.

The cases discussed indicate that to a first approximation it is usually sufficient to replace even a rectangular distribution within a link by its equivalent or best-fitting Gaussian distribution, and then add variances to obtain the variance of the overall distribution, which should also be regarded as Gaussian. This procedure involves much less calculation than is explained here, but no better approximation is justified unless there is a great improvement in the data available concerning the actual distribution of errors within individual links and of overall errors.

1. INTRODUCTION

The major distortions that are introduced by equipment in the colour television chain are known and the amounts of them that are tolerable are likewise known. Although adequate technical information is still lacking, the overall errors in chrominance signal differential phase and gain have been analysed[1] on the assumption that the error contributed by any link in the chain is equally likely to have any value within certain (equal and opposite) limits appropriate to that link.

It is assumed in what follows that the individual distributions are rectangular and that errors can be added linearly. Although in some cases these two assumptions may be open to objection, from the engineering point of view, they do permit a definite prediction of overall errors to be relatively easily calculated. Such a prediction is often, as here, an essential preliminary to the formulation of more adequate but probably more complicated assumptions.

If a suitable degree of quantization of the values of the errors contributed by each link is permitted, this analysis is exact, though somewhat tedious when several links are considered, as is usually necessary. The analysis gives the probability that any given overall deviation shall be exceeded.

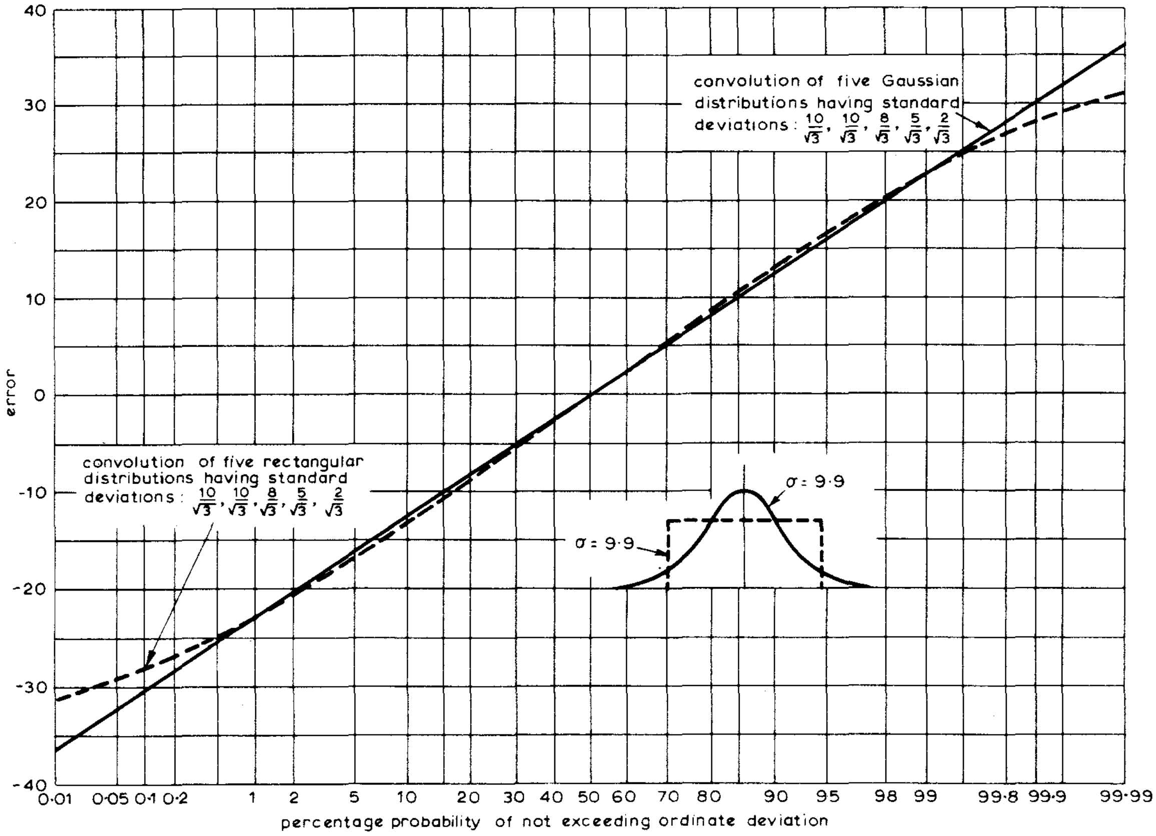

Fig. 1 - Cumulative probability of occurences of overall errors from five devices in tandem

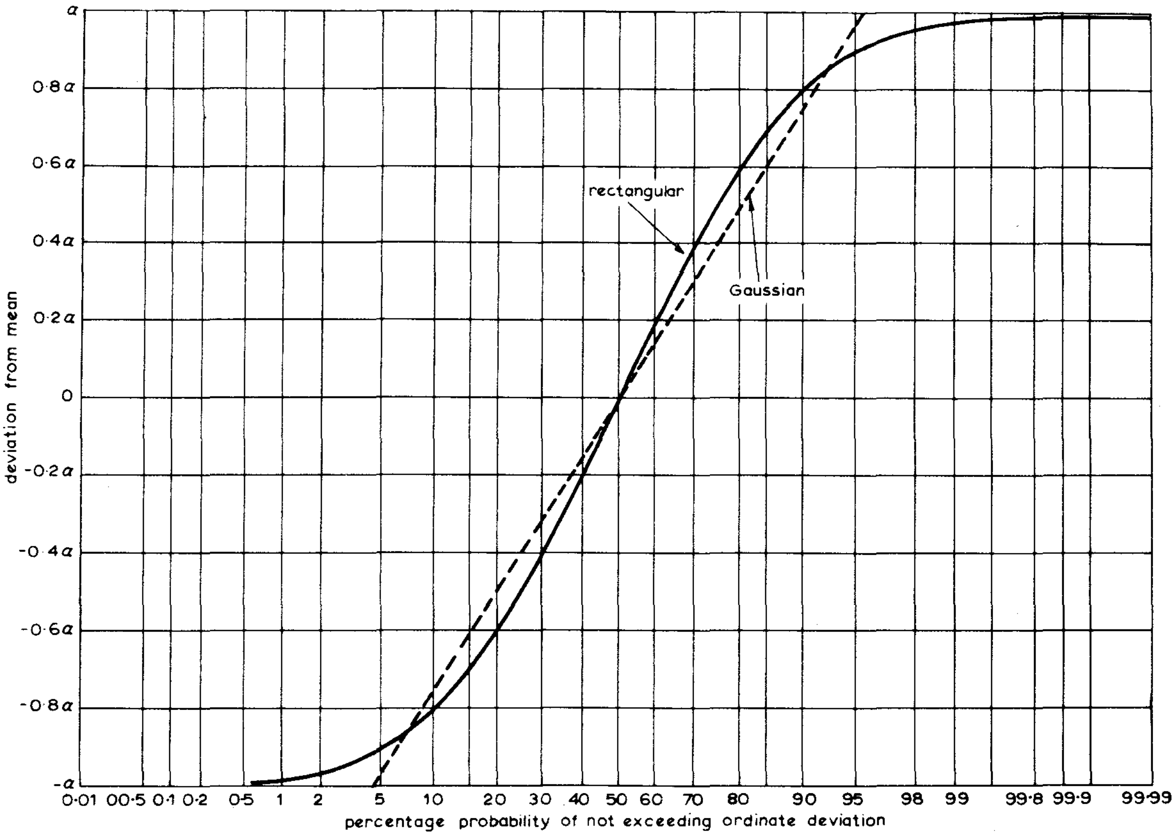

Fig. 2 - Comparison of rectangular distribution and its 'Gaussian Equivalent.' (Rectangle has any value between ± α equally likely)

Another possible method of obtaining the probability that any given overall deviation be exceeded is by replacing each rectangular distribution between limits $\pm\alpha_n$, say, of the error contributed by a link by the corresponding best-fitting normal or Gaussian distribution, which has mean zero and standard deviation $\alpha_n/\sqrt{3}$. When this is done, the distribution of the overall deviation is obtained immediately as a Gaussian distribution whose variance is the sum of the variances of the individual links. Fig. 4 of reference 1 (reproduced for convenience as Fig. 1 here) shows that there is very little difference between results obtained by the two methods when there are five constituent links having individual limits ± 10, ± 10, ± 8, ± 5 and ± 2. By contrast, Fig. 2 shows that a single rectangular distribution differs significantly from the corresponding best-fitting Gaussian distribution.

In Section 2, the combined effect of up to five equal quantized rectangular distributions is calculated for a coarse quantization of ten elements only. The effect of doubling the number of quantized elements is considered in Section 3, and this process can be repeated as often as desired. A simple procedure for changing from equal to unequal quantized rectangular distributions is considered in Section 4. Although this combination process of 'convolution' can be carried out for a continuous distribution, the use of sufficiently fine quantization appears to be preferable (except in the case of a Gaussian distribution, for which 'convolution' is equivalent to addition of variances as already mentioned).

2. CONVOLUTION OF EQUAL RECTANGULAR DISTRIBUTIONS QUANTIZED TO TEN ELEMENTS

Consider the rectangular distribution of x of Fig. 3, which is quantized to ten elements, so that we regard any value between $-\alpha$ and $-0{\cdot}8\alpha$ (within the left-hand shaded rectangle) as replaceable by $-0{\cdot}9\alpha$, and any value between $-0{\cdot}8\alpha$ and $-0{\cdot}6\alpha$ (within the white area next to the right) as equivalent to $-0{\cdot}7\alpha$, and so on. It is convenient to work in terms of the 'generating function' (reference 2) associated with this distribution, which is

$$f_1(X) = \frac{1}{10}\Big[X^{-4\cdot5} + X^{-3\cdot5} + X^{-2\cdot5} + X^{-1\cdot5} + X^{-0\cdot5} + X^{0\cdot5} + X^{1\cdot5} + X^{2\cdot5} + X^{3\cdot5} + X^{4\cdot5}\Big] \tag{1}$$

$$= \frac{1}{10} X^{-4\cdot5}(X^{10} - 1)/(X-1) \tag{2}$$

At this stage Equation (1) can be regarded merely as a kind of shorthand language for conveying succinctly the same information as is conveyed geometrically by Fig. 3. The first term can be read as equivalent to saying: 'Fig. 3 has a rectangle (at the left-hand end) whose centre is 4·5 fifths of α to the left of the origin 0 (because the index of X is $-4{\cdot}5$), and whose area is 0·1 (because the coefficient of $X^{-4\cdot5}$ is 0·1).' The next part of Fig. 3 is a rectangle also of area 0·1, but its centre is 3·5 fifths of α to the left of 0, and so on. All the rectangles are of equal width $0{\cdot}2\alpha$ because of the above-mentioned quantization. The heights of the rectangles are equal in the case of Fig. 3 because all values between $-\alpha$ and $+\alpha$ are equally likely, but they would in general vary with the relative probability of values of x in the ten quantized sections. In this language X^n can be interpreted as an instruction to shift the rectangle of width $0{\cdot}2\alpha$ and area equal to the coefficient of X^n so that its centre is $0{\cdot}2n\alpha$ to the right of 0. Now the reason why Equation (1) is a useful idea is that if we have two distributions expressed in this way, x in terms of $f_1(X)$ as in Equation (1) and y in terms of a similar expression $f_2(X)$, then the corresponding expression for the distribution $x + y$ happens to be $f_1(X)f_2(X)$ where the multiplication is carried out as if X were an ordinary algebraic number (see reference 2). In principle $f_1(X)$, $f_2(X)$ etc. are of the form (polynomial in X) multiplied by a power of X which may not be integral — it is not in Equation (1). Any algebraic manipulation, such as going from Equation (1) to Equation (2), is to be considered legitimate, because it is essentially reversible.

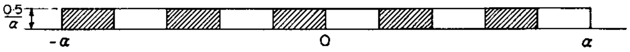

Fig. 3 - Geometrical representation of a rectangular distribution of width ± α, quantized to ten elements

It follows that the distribution of the sum of two equal distributions like that in Fig. 3 has the generating function $\{f_1(X)\}^2$, which is preferably expressed in terms of Equation (2) as

$$\Big\{f_1(X)\Big\}^2 = \frac{1}{100} X^{-9}(1-X^{10})^2(1-X)^{-2} \tag{3}$$

Now from Fig. 3 we see that the lowest possible power of X is -9, and this only occurs if both distributions are in their lowest sections; likewise the highest possible power of X is $+9$ which only occurs when both distributions are in their highest sections. We therefore know that, despite the fact that there is a factor $(1 - X)^{-2}$, which appears to lead to an infinite series, in Equation (3), the series will in fact terminate.

If $-9 \leqslant N \leqslant 0$, the coefficient of X^N in (3) is obtainable by replacing the numerator factor $(1-X^{10})^2$ by 1, and is $N + 10$. For $1 \leqslant X \leqslant 10$, however, the numerator factor $(1-X^{10})^2$ must be replaced by $1-2X^{10}$. As before, the first or 1 term of this factor gives a coefficient of $N+10$, but the $-2X^{10}$ term of this factor contributes a coefficient of $-2N$, so that the total is $(10-N)$. Finally, when $N \geqslant 11$, the last term X^{20} of the numerator makes a contribution $(N-10)$ to the coefficient of X^N, so that the total result is zero. Thus we end up with the form

$$\left\{f_1(X)\right\}^2 = \frac{1}{100}\left[X^{-9} + 2X^{-8} + 3X^{-7} . \; . \; . \; . + 9X^{-1} + 10 + 9X + 8X^2 + . \; . \; . \; . + X^9\right] \tag{4}$$

which could have been derived also, but more tediously, from Equation (1) by direct algebraic multiplication, and has the geometrical representation shown in Fig. 4 (analogous to Fig. 3). The diagram on probability paper analogous to Fig. 1 is Fig. 5, from which it is seen that the discrepancy between the distribution derived from (4) and the equivalent Gaussian is markedly reduced.

The corresponding expression for convolving three equal rectangular distributions is

$$\begin{aligned}\left\{f_1(X)\right\}^3 = 10^{-3}\Big[&X^{-13\cdot5} + 3X^{-12\cdot5} + 6X^{-11\cdot5} + 10X^{-10\cdot5} + 15X^{-9\cdot5} + 21X^{-8\cdot5}\\ &+ 28X^{-7\cdot5} + 36X^{-6\cdot5} + 45X^{-5\cdot5} + 55X^{-4\cdot5} + 63X^{-3\cdot5} + 69X^{-2\cdot5}\\ &+ 73X^{-1\cdot5} + 75X^{-0\cdot5} + 75X^{0\cdot5} + 73X^{1\cdot5} + 69X^{2\cdot5} + 63X^{3\cdot5}\\ &+ 55X^{4\cdot5} + 45X^{5\cdot5} + 36X^{6\cdot5} + 28X^{7\cdot5} + 21X^{8\cdot5}\\ &+ 15X^{9\cdot5} + 10X^{10\cdot5} + 6X^{11\cdot5} + 3X^{12\cdot5} + X^{13\cdot5}\Big]\end{aligned} \tag{5}$$

and that for $\{f_1(X)\}^4$ is best given in the form

$$10^{-4}\sum a_{4r}X^{-18+r}$$

where a_{4r} is given in Table 1 and $\{f_1(X)\}^5$ is best given in the form

$$10^{-5}\sum a_{5s}X^{-22\cdot5+s}$$

with a_{5s} as in Table 2.

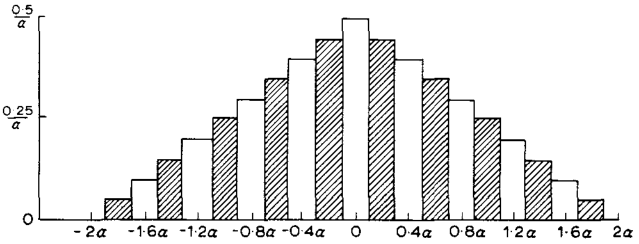

Fig. 4 - Geometrical representation of the sum (by convolution) of two identical distributions represented by Fig. 3

Thus we have obtained generating functions, exact within the limits of quantization, for the convolution of up to five equal rectangular distributions, from which histograms analogous to Figs. 3 and 4 can be derived. It remains to consider (in Section 3) what happens, in the case of rectangular distributions, if the quantization interval is halved (a process which could be repeated as often as required to obtain any desired fineness of quantization). We must also consider (in Section 4) the surprisingly simple modifications required when the rectangular distributions to be convolved are not equal.

TABLE 1

Values of a_{4r} *when* $\left\{f_1(X)\right\}^4 = 10^{-4}\sum a_{4r}X^{-18+r}$

r	0	1	2	3	4	5	6	7	8	9
a_{4r}	1	4	10	20	35	56	84	120	165	220
r	10	11	12	13	14	15	16	17	18	19
a_{4r}	282	348	415	480	540	592	633	660	670	660
r	20	21	22	23	24	25	26	27	28	29
a_{4r}	633	592	540	480	415	348	282	220	165	120
r	30	31	32	33	34	35	36	Over 36		
a_{4r}	84	56	35	20	10	4	1	0		

TABLE 2

Values of a_{5s} *when* $\left\{f_1(X)\right\}^5 = 10^{-5}\sum a_{5s}X^{-22\cdot5+s}$

s	0	1	2	3	4	5	6	7	8	9
a_{5s}	1	5	15	35	70	126	210	330	495	715
s	10	11	12	13	14	15	16	17	18	19
a_{5s}	996	1340	1745	2205	2710	3246	3795	4335	4840	5280
s	20	21	22	23	24	25	26	27	28	29
a_{5s}	5631	5875	6000	6000	5875	5631	5280	4840	4335	3795
s	30	31	32	33	34	35	36	37	38	39
a_{5s}	3246	2710	2205	1745	1340	996	715	495	330	210
s	40	41	42	43	44	45	Over 45			
a_{5s}	126	70	35	15	5	1	0			

3. HALVING THE QUANTIZATION INTERVAL

If in Fig. 3 the quantization interval is halved, each of the ten rectangles of width $0{\cdot}2\alpha$ and height $0{\cdot}5/\alpha$ must be divided into two equal rectangles of width $0{\cdot}1\alpha$ and height $0{\cdot}5/\alpha$, so that the expression $f_2(X)$ analogous to $f_1(X)$ in Equation (1) for the generating function is

$$f_2(X) = 0{\cdot}05 \Big[X^{-4\cdot75} + X^{-4\cdot25} + X^{-3\cdot75} + X^{-3\cdot25} + X^{-2\cdot75} + X^{-2\cdot25} + X^{-1\cdot75} + X^{-1\cdot25} + X^{-0\cdot75} + X^{-0\cdot25} + X^{0\cdot25} + X^{0\cdot75} + X^{1\cdot25} + X^{1\cdot75} + X^{2\cdot25} + X^{2\cdot75} + X^{3\cdot25} + X^{3\cdot75} + X^{4\cdot25} + X^{4\cdot75}\Big] \qquad (6)$$

which can be written

$$f_2(X) = \tfrac{1}{2}(X^{-0\cdot25} + X^{0\cdot25})f_1(X) \qquad (7)$$

so that $\{f_2(X)\}^n$ can be obtained by algebraic multiplication of $\{f_1(X)\}^n$ by the relatively simple expression

$$Z^n = \frac{1}{2^n}\,(X^{-0\cdot25} + X^{0\cdot25})^n \qquad (8)$$

We shall derive fully only $\{f_2(X)\}^3$; the procedure is perfectly general. We have

$$Z^3 = \frac{1}{8}\Big[X^{-0\cdot75} + 3X^{-0\cdot25} + 3X^{0\cdot25} + X^{0\cdot75}\Big] \qquad (9)$$

so that the derivation of the coefficients of various powers of X in 8000 $\{f_2(X)\}^3$ can be derived from Equations (5) and (9) as set out in Table 3, which is carried on only far enough to ensure that the symmetry of the result to be expected from the symmetry of the factors Z^3 in (9) and $\{f_1(X)\}^3$ in (5) is seen to be established.

This calculation indicates clearly that retrospective refinement of the quantization is relatively easy, and that a coarse quantization is required initially, since the calculation is then quite straightforward.

4. CONVOLUTION OF TWO OR MORE UNEQUAL RECTANGULAR DISTRIBUTIONS

Let us consider next the convolution of two distributions, one of which is that of Fig. 3, while the other is a rectangular distribution extending from $-n\alpha/10$ to $+n\alpha/10$ where n is an integer; as in Fig. 3 we shall suppose this distribution to be quantized into sections of width $0{\cdot}2\alpha$, so the height of the rectangles analogous to those of Fig. 3 for the second distribution will be $5/n\alpha$. The generating function $f_2(X)$ for this second distribution is given by

$$f_3(X) = \frac{1}{n}\Big[X^{-\frac{1}{2}(n-1)} + X^{-\frac{1}{2}(n-3)} + X^{-\frac{1}{2}(n-5)} + \ldots\ldots + X^{\frac{1}{2}(n-5)} + X^{\frac{1}{2}(n-3)} + X^{\frac{1}{2}(n-1)}\Big] \qquad (10)$$

so that if n is even, the origin will separate two of the quantized rectangles as in Fig. 3, whereas if n is odd, these will be a central rectangle of width $0{\cdot}2\alpha$ which is half on the left of the origin and half on the right.

It is clear from Equations (10) and (1) that the lowest power of X occurring in the product $f_1(X)f_3(X)$ is $X^{-\frac{1}{2}(n+8)}$ and that the highest power of X involved is $X^{\frac{1}{2}(n+8)}$. The expression $f_1(X)f_3(X)$ is also symmetrical, so that the coefficient of X^r is the same as that of X^{-r}. By analogy with Equation (2), however, we can write

$$f_1(X)f_3(X) = \frac{1}{10n}\,X^{-\frac{1}{2}(n+8)}\,(1-X^{10})\,(1-X^n)/(1-X)^2 \qquad (11)$$

but since we already know $\{f_1(X)\}^2$ from Equation 4, for purposes of calculation it is preferable to write

$$f_1(X)f_3(X) = \frac{10}{n}\,X^{-\frac{1}{2}(n-10)}\Big\{f_1(X)\Big\}^2(1-X^n)(1+X^{10}+X^{20}+\ldots.) \qquad (12)$$

The case $n = 4$ is shown in Table 4, and leads us to a general inference that if $n \leqslant 10$, the coefficient of $X^{-\frac{1}{2}(n+8)+r}$ in $10nf_1(X)f_3(X)$ is $r+1$ for $0 \leqslant r \leqslant n-1$ but remains constant and equal to n for $n-1 \leqslant r \leqslant \frac{1}{2}(n+8)$. The remaining coefficients belong to positive powers of X and are therefore obtainable by symmetry. If, however, $n \geqslant 10$, the coefficient of $X^{-\frac{1}{2}(n+8)+r}$ is $r+1$ for $0 \leqslant r \leqslant 9$, and remains constant and equal to 10 for $9 \leqslant r \leqslant \frac{1}{2}(n+8)$. If n is much less than 10, $f_1(X)f_3(X)$ only differs from $f_1(X)$ when r is near the extreme limits 0 and $n+8$; if n is much greater than 10, $f_1(X)f_3(X)$ only differs from $f_3(X)$ when r is near these extreme limits. If $n < 10$ but $(10-n)$ is small, $f_1(X)f_3(X)$ only differs from $(10/n)\{f_1(X)\}^2$ for small powers of X; if $n > 10$ but $(n-10)$ is small, $f_1(X)f_3(X)$ only differs from $(n/10)\{f_3(X)\}^2$ for small powers of X.

The probability of extreme values of r is controlled mainly by the product mn when a rectangular distribution of width $\pm 0{\cdot}2m\alpha$ is convolved with one of width $\pm 0{\cdot}2n\alpha$. If, therefore, $(m+n)$ is constant and $m \approx n$, it is the combined width of the distributions that controls probabilities of extreme values, but the width of the wider distribution which controls probabilities near the mean.

TABLE 3

Derivation of Coefficients of 8000 $\{f_2(X)\}^3$ from Algebraic Multiplication of Equations (5) and (9)

$X^{n/4}$: Value of n	−57	−55	−53	−51	−49	−47	−45	−43	−41	−39	−37	−35	−33	−31	−29	−27	−25	−23	−21	−19	−17	−15	−13	−11	−9	−7	−5	−3	−1	1	3	5	7	9	11	13	15
$YX^{-0\cdot75}$	1		3		6		10		15		21		28		36		45		55		63		69		73		75		75		73		69		63		55
$3YX^{-0\cdot25}$		3		9		18		30		45		63		84		108		135		165		189		207		219		225		225		219		207		189	
$3YX^{0\cdot25}$			3		9		18		30		45		63		84		108		135		165		189		207		219		225		225		219		207		189
$YX^{0\cdot75}$				1		3		6		10		15		21		28		36		45		55		63		69		73		75		75		73		69	
Total: $8YZ^3$	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	171	190	210	228	244	258	270	280	288	294	298	300	300	298	294	288	280	270	258	244

$Y = 1000\{f_1(X)\}^3$; tabular entry under n is the coefficient of $X^{n/4}$ in the expression on the left of the same row.

TABLE 4

Evaluation of $f_1(X)\,f_3(X)$ when $n = 4$

r	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$100X^3\ \{f_1(X)\}^2$	1	2	3	4	5	6	7	8	9	10	9	8	7	6	5	4	3	2	1						
$-100X^7\ \{f_1(X)\}^2$					−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1		
$100X^{13}\ \{f_1(X)\}^2$											1	2	3	4	5	6	7	8	9	10	9	8	7	6	5
$-100X^{17}\ \{f_1(X)\}^2$															−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−9
$100X^{23}\ \{f_1(X)\}^2$																					1	2	3	4	5
$-100X^{27}\ \{f_1(X)\}^2$																									−1
$40\,f_1(X)f_3(X)$	1	2	3	4	4	4	4	4	4	4	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0

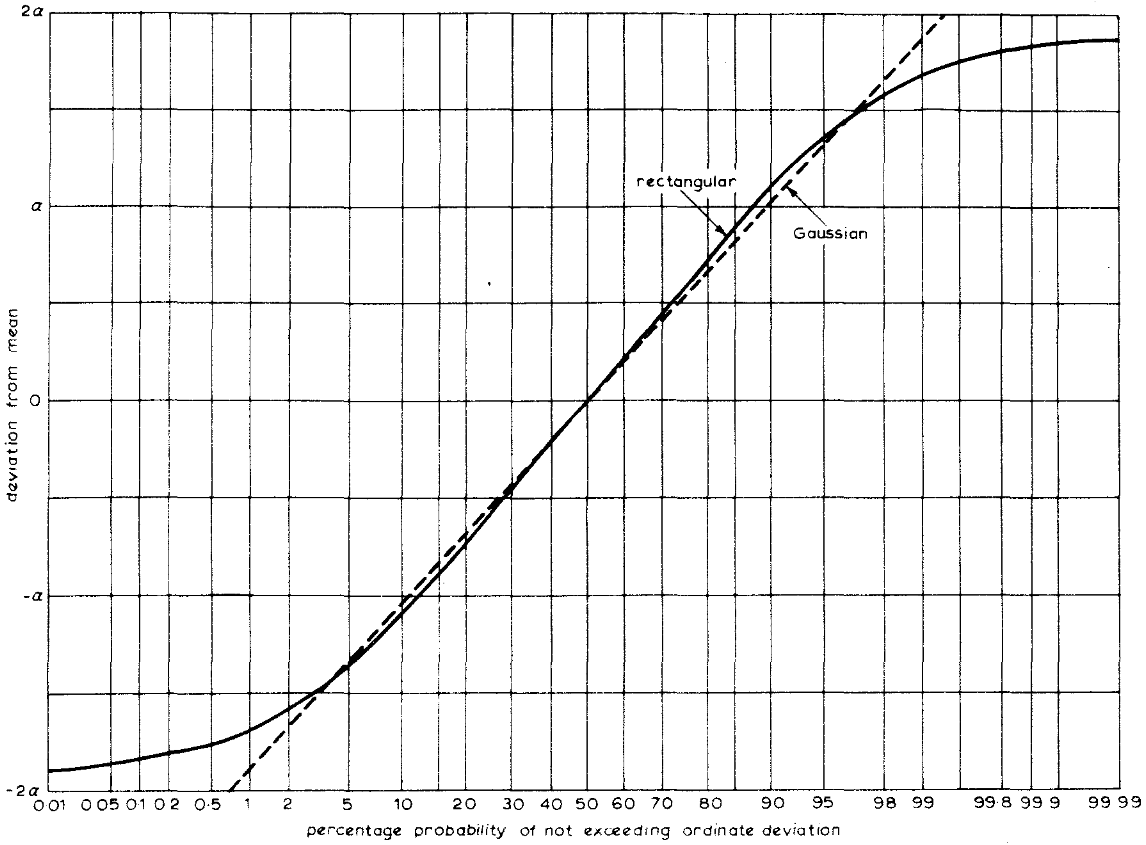

Fig. 5 - Comparison of the convolution of two equal rectangular distributions and the 'Gaussian Equivalent'

When several distributions have to be convolved instead of only two, the extension of the procedure used to obtain Table 4 is reasonably straightforward. What is required is the addition of a moderate number of displaced multiples of the sets of coefficients given in Equations (4) and (5) or in Tables 1 and 2.

Now a comparison of Fig. 5 with Fig. 2 shows that the convolution of even two equal rectangular distributions does not differ greatly from the equivalent or best-fitting Gaussian distribution. Fig. 1 also indicates that the same is true for the convolution of five rectangular distributions of widths ± 10, ± 10, ± 8, ± 5 and ± 2 arbitrary units.

5. DISCUSSION OF RESULTS

The calculations carried out in Sections 2, 3 and 4 permit us to determine the probability that any given overall error will be exceeded when that overall error is due to the linear addition of a number of errors each distributed rectangularly. The only approximation involved is the quantization, and in Section 3 it was shown how this could in effect be easily refined for rectangular distributions. It was also noted that in Fig. 1 the error committed by replacing each of several rectangular distributions by the equivalent or best-fitting Gaussian distribution was unimportant except at very high overall error values.

If the distribution within a link is not rectangular, but quantization is permitted, it is still possible to obtain the 'generating function' of the overall distribution in a form analogous to Equations (4) or (5) by direct algebraic multiplication, but in general there will be no simplified form of the overall 'generating function' analogous to Equations (3) or (7). It will also still be possible to replace the actual distribution for each link by the equivalent or best-fitting Gaussian distribution, and add variances as before; in the author's opinion, this is the best procedure.

It has also been suggested that the proper criterion as to whether the overall distortion of a particular kind is tolerable or not should be expressed in terms of say the rth power of the individual distortion components. This means that the rth powers of the maximum distortions contributed by the individual links are added, and the rth root of the sum, say X, is taken as the appropriate measure of overall distortion. If the actual distribution is replaced by the equivalent best-fitting Gaussian distribution for each link, the effective value of r is 2.

It is easy enough to decide whether X is greater than or less than a given limit, but it is not easy to determine, for a general value of r, the percentage of time for which X is likely to exceed the limit. The percentage of time for which any given overall limit is exceeded can be determined

(a) if the distribution for each link is replaced by the equivalent or best-fitting Gaussian distribution

(b) if the distribution for each link is regarded as rectangular and suitably quantized

(c) if the quantized distribution for each link is known

The determination is easiest in case (a) and has been discussed above for case (b); it is possible but tedious in case (c). As already mentioned, the information available as to the actual probability distribution of errors contributed by each link is at present far from adequate, and it is important that every possible opportunity of remedying this situation should be taken.

Now a single rectangular distribution differs appreciably from the equivalent Gaussian distribution, as shown in Fig. 2. The probability that the rectangular distribution has a deviation from the mean numerically greater than α is zero. This means that, on arithmetic probability paper, the rectangular distribution curve must have horizontal asymptotes at deviations $\pm\alpha$, whereas the Gaussian distribution is represented by a straight line. The probability of exceeding a value $0{\cdot}5\alpha$, for example, is 0·25 in the rectangular case but 0·20 in the equivalent Gaussian case. These discrepancies, however, are very much less in the case when two equal rectangular distributions are convolved, as in Fig. 5, and very small indeed (except for very large overall errors) in the composite case considered in Fig. 1.

If a rectangular distribution is convolved with another of much smaller width, the resultant only differs from the original wider distribution at extreme values. If, however, two or more rectangular distributions of nearly equal widths are convolved, the resultant can be regarded as nearly Gaussian.

In general, probability distributions of a single variate which differ from the equivalent or best-fitting normal distribution as much as a rectangular distribution does appear to be rare. It can be argued that, in the case of the phase and differential-gain errors discussed here, there is a tendency for values near the permitted limits to predominate,* on the ground that people controlling the links will not wish to take corrective action until they are compelled to do so by the appearance of a near-limit value. But in the absence of much stronger evidence than is at present available concerning the actual distribution of errors within links, the assumption made here that errors within links are distributed rectangularly appears to be the best available. At least it permits a prediction of overall errors to be made. By testing such predictions against observed results, it should be possible to obtain better information about the actual distribution of errors within each link. In the author's opinion, however, the overall error distribution is unlikely to be very sensitive to changes in the assumed form of error distribution within each link made necessary by better information. In other words, useful and reliable predictions of the percentage of time for which the overall error exceeds a given limit can be made as if the distribution within each link is either rectangular or the best-fitting Gaussian equivalent thereof.

6. CONCLUSIONS AND RECOMMENDATIONS

It is therefore suggested that the best procedure for assessing the overall distribution of phase and differential gain errors in television distribution chains is to proceed as if the error distribution within any one link is rectangular, obtain the best-fitting or equivalent Gaussian distribution for each link of the chain, and then add variances to obtain the overall variance and treat the overall distribution as Gaussian.

This gives a result which is 5% more optimistic than that obtained by simple root-sum-of-squares addition, which is the most optimistic of the three laws used by the CMTT, (Mixed Television Transmission Committee) but as it is so easily calculated, an empirical addition of 10% is suggested as an adequate remedy for over-optimism.

Thus although the calculations explained in this report are reasonably straightforward, and are exact within the limits of the quantization, from the practical point of view the primary objective is to show that such calculations are frequently unnecessary.

Replacing any actual distribution by its equivalent Gaussian distribution will usually be an adequate first approximation, and the only approximation justified by our present very scanty knowledge of the actual nature of the distributions within links. If, however, later information indicates that a rectangular distribution is in fact a better approximation than the best-fitting Gaussian one to the actual distribution of errors within the principal links, the procedures discussed above will permit a second approximation to be obtained quite easily.

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* 'Cow-horn' distribution

No significant progress can be made, however, without adequate data on both actual errors within individual links and actual overall errors in complete chains, and the main recommendation must therefore be the provision of adequate actual operational data.

7. REFERENCES

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Printed by BBC Research Department, Kingswood Warren, Tadworth, Surrey